Lesson #62 · Topic 4F · Practice

Answer Key

1. a. True; since the standardized test statistic is negative, $\bar{x}_1 - \bar{x}_2$ must be negative, which means $\bar{x}_1 < \bar{x}_2$.
   b. Not enough information; the sample standard deviations are unknown.
   c. False; the p -value is 0.0761 which is greater than 0.05. The results are not statistically significant so there is not convincing evidence for H_a .
   d. True; the p -value is 0.0381 which is less than 0.05. The results are statistically significant so there is convincing evidence for H_a .

2. a. Type of study: Observational study
   Variables: Store (Nantucket Bakery or Hall Street Bakery) (categorical), freshness score for each loaf of bread (quantitative)
   b. Two-sample t -test for $\mu_N - \mu_H$, α = 0.05
   $\mu_N - \mu_H$ = true difference in mean freshness scores (Nantucket $-$ Hall Street)
   $H_0 : \mu_N - \mu_H = 0$
   $H_a : \mu_N - \mu_H \neq 0$
   c. Random: independent random samples of 30 loaves of bread from Nantucket and 25 loaves of bread from Hall Street ✓
   10%: 30 $\leq$ 0.10 (all loaves made by Nantucket), 25 $\leq$ 0.10 (all loaves made by Hall Street) ✓
   Normal/Large Sample: For Hall Street: The distribution of freshness scores had no strong skewness or outliers and for Nantucket bakery: 30 $\geq$ 30 (CLT) ✓
   d. $t = \dfrac{(\bar{x}_N - \bar{x}_H) - (\mu_N - \mu_H)}{\sqrt{\dfrac{s_N^2}{n_N} + \dfrac{s_H^2}{n_H}}} = \dfrac{(9.2 - 8.5) - 0}{\sqrt{\dfrac{1.5^2}{30} + \dfrac{1.2^2}{25}}} = 1.922, \mathrm{df} = 24$, p -value = 0.067

   (using technology: df = 52.93, p -value = 0.060)
   e. Because 0.067 > 0.05, we fail to reject H_0 and we do *not* have convincing statistical evidence that the mean freshness score for all loaves from Nantucket Bakery is different from the mean freshness score for all loaves from Hall Street Bakery.

3. a. CLASSFIY:

Type of study: Observational study

Variables: Age of American (30-49 years old or 65+ years old) (categorical), amount of tip left (quantitative)

CHOOSE:

two-sample t -test for $\mu_1 - \mu_2$ at $\alpha = 0.05$

$\mu_1 - \mu_2 =$ true difference in mean tip percentages left for a moderate dining experience at a restaurant for Americans (aged 65+ – aged 30-49)

$H_0 : \mu_1 - \mu_2 = 0; H_a : \mu_1 - \mu_2 < 0$

CHECK:

<u>Random</u>: independent random samples of 2,007 Americans aged 65+ and 3,116 Americans aged 30-49 ✓

<u>10%</u>: $2,007 \leq 0.10$ (all Americans aged 65+), $3,116 \leq 0.10$ (all Americans aged 30-49) ✓

<u>Normal/Large Sample</u>: $2,007 \geq 30$, $3,116 \geq 30$; central limit theorem ✓

CALCULATE:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(15.76 - 16.4) - 0}{\sqrt{\frac{3.61^2}{2,007} + \frac{4.079^2}{3,116}}} = -5.88, \mathrm{df} = 2,006, \ p\text{-value} \approx 0$$

(using technology, $\mathrm{df} = 4,640.75$ and p -value $= 2.15 \times 10^{-9}$)

CONCLUDE:

Because 0.00000000215 < 0.05, we reject H_0 and we do have convincing statistical evidence that Americans aged 65+ tip less than Americans aged 30-49 for a moderate dining experience at a restaurant, on average.

b. Since H_0 was rejected, a Type I error could have been made. This means that we concluded that the Americans aged 65+ tip less than Americans aged 30-49, when in reality they tip the same.

4. A one-sided test $(H_0 : \mu_1 = \mu_2$ and $H_a : \mu_1 > \mu_2)$ is more likely to result in a rejection of the null hypothesis. In a two-sided test we consider the proportion of values in the sampling distribution that are greater than or equal to our test statistic and less than or equal to the opposite signed test statistic, resulting in a p -value that is double the p -value of the one-sided test when we have some evidence for the alternative hypothesis $(\bar{x}_1 > \bar{x}_2)$. Since we reject the null hypothesis when the p -value is small, the one-sided test would be more likely to reject the null hypothesis.

5. B

6. B