Lesson #55 · Topic 4B · Practice

Answer Key

1. a. Random – The Random condition makes the sample mean an unbiased estimator of the population mean and allows us to generalize results to the population of interest.

   10% – The 10% condition allows us to consider individual observations independent of each other when sampling without replacement, so we can use the standard error formula for the sampling distribution of $\bar{x}$.

   Normal/Large Sample – The Normal/Large Sample condition ensures that the sample size is large enough for the central limit theorem to apply, making the sampling distribution of $\bar{x}$ approximately normal regardless of the shape of population distribution.

   b. Random: random sample of 35 AP Exam scores ✓
   10%: $35 \leq 0.10$ (all 356 AP Exam scores from this school) ✓
   Normal/Large Sample: $35 \geq 30$, central limit theorem ✓

2. a. $n = 20$ will have the largest t^* because as the sample size increases, the t^* decreases.

   b. For $n = 20, df = 19, t^* = 1.729$
   For $n = 30, df = 29, t^* = 1.699$
   For $n = 40, df = 30, t^* = 1.697$

   c. For $n = 20, df = 19, t^* = 1.729$
   For $n = 30, df = 29, t^* = 1.699$
   For $n = 40, df = 39, t^* = 1.685$

   For $n = 20$ and $n = 30$, Table B and technology are the same. For $n = 40$, Table B gives a slightly higher t^* because $df = 39$ is not available and $df = 30$ needs to be used.

   d. Since the population standard deviation is unknown and Shruthi must use the sample standard deviation to estimate it, she will use a critical value of t^* to account for the additional source of variability. The t^* value is always larger than the corresponding z^* value to account for this additional variability.

3. a. $df = 9, \; t^* = 2.821$

   b. $SE = \dfrac{5.896}{\sqrt{10}} = 1.864, \; ME = 2.821(1.846) = 5.260$

   c. $30.1 \pm 5.26 = (24.84, 35.36)$

   d. The interval calculated in part (c) includes the value of 35, so it is plausible that the true mean number of goals scored by the top 100 players is 35.

4. a. The Normal/Large Sample condition is not met because the sample size is less than 30, and the distribution of the sample is strongly skewed with one outlier.

   b. Yes, $40 \geq 30$, so the central limit theorem applies.

   c. If all else remains the same, a larger sample size will

5. a. $\mu =$ true mean Rotten Tomatoes score for all Netflix original movies released in recent years

   b. Random: 30 randomly selected movies ✓

   10%: $30 \leq 0.10$ (all Netflix original movies released in recent years) ✓

   Normal/Large Sample: $30 \geq 30$, central limit theorem ✓

   c. $df = 29, t^{*} = 2.045, ME = 2.045\left(\frac{6.5}{\sqrt{30}}\right) = 2.427$

   d. $62 \pm 2.427 = (59.57, 64.43)$

   We are 95% confident the interval from 59.57 to 64.43 captures the true mean Rotten Tomatoes score for all Netflix original movies released in recent years.

   e. No, the interval calculated does not include 65, so there is convincing statistical evidence that the true mean Rotten Tomatoes score for Netflix is not 65.

6. A

7. C