Lesson #48 · Topic 3G · Practice

Answer Key

1. a. p_1 and p_2 are equal.

   b. $\hat{p}_c = \dfrac{51+61}{150+100} = \dfrac{112}{250} = 0.448$. $\hat{p}_c$ is the sample proportion of people who would say social media has been a good thing for democracy when we combine (or "pool") the data from the two samples as if they came from one larger sample.

   c. $150(0.448) = 67.2,\ \ 150(1-0.448) = 82.8,\ \ 100(0.448) = 44.8,\ \ 100(1-0.448) = 55.2,$ which are all ≥ 10 .

   d. $\sqrt{\dfrac{(0.448)(0.552)}{150} + \dfrac{(0.448)(0.552)}{100}} = 0.0642$

   e. In a two-sample z -test for $p_1 - p_2$, we assume that the population proportions are equal. $\hat{p}_c$ is our best estimate for what the population proportions would be if they were equal.

2. a. Yes. The Pew Research Center selected two separate random samples, which is required to obtain samples that are representative of their respective populations, so they can generalize the results to the populations of all adults in 2022 and 2023.

   b. Yes, $1,000 \leq 0.10$ (all adults in 2022) and $2,000 \leq 0.10$ (all adults in 2023). When sampling without replacement, the 10% condition allows for observations to be viewed as independent of each other so that we can use the formula for standard error.

   c. $\hat{p}_c = \dfrac{380+1040}{1000+2000} = 0.473,\ \ 1,000(0.473) = 473,\ \ 1,000(0.527) = 527,\ \ 2,000(0.473) = 946,$ and $2,000(0.527) = 1,054$ are all ≥ 10. This is so we can confirm the sampling distribution of $\hat{p}_{22} - \hat{p}_{23}$ is approximately normal.

3. a. Type of study: Experiment
   Variables: Type of teaching method (categorical), whether the student has a grade of C or higher (categorical)

   b. Two sample z -test for $p_1 - p_2$ at α = 0.05
   $p_1 - p_2$ = the true difference in proportions of students like these who would receive a grade of C or higher on the post-test (lecture $-$ small group).
   $H_0 : p_1 - p_2 = 0, H_a : p_1 - p_2 \neq 0$

   c. $\underline{\text{Random:}}$ Students were randomly assigned to either lectures or collaborative small groups.
   $\underline{10\%:}$ Not needed since we are not sampling without replacement.
   $\underline{\text{Large Counts:}}$ 75(0.544) = 40.8; 75(0.456) = 34.2; 72(0.544) = 39.2; and 72(0.456) = 32.8 are all $\geq$ 10.

   d. $$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\hat{p}_c(1-\hat{p}_c)}{n_1} + \dfrac{\hat{p}_c(1-\hat{p}_c)}{n_2}}} = \frac{(0.440 - 0.653) - 0}{\sqrt{\dfrac{(0.554)(0.446)}{75} + \dfrac{(0.554)(0.446)}{72}}} = -2.59$$
   p -vale = 0.0096

   e. Assuming H_0 is true $(p_1 - p_2 = 0)$, there is a 0.0096 probability of getting a $\hat{p}_1 - \hat{p}_2$ of -0.213 or more extreme in either direction purely by chance. Because 0.0096 < 0.05, we reject H_0 and do have convincing statistical evidence of a difference in proportions of students like these who would receive a grade of C or higher on the post-test (lecture $-$ small group).

   f. No since the hypothesis test rejected $H_0 : p_1 - p_2 = 0$, at $\alpha = 0.05$ level, the corresponding 95% confidence interval would not contain 0 as a plausible value.

4. CLASSIFY:
Type of study: Observational study
Variables: Whether sophomores at a large high school purchase school lunch (categorical); whether freshman at a large high school purchase school lunch (categorical)

CHOOSE:
two-sample z -test for $p_f - p_s$ at $\alpha = 0.05$
$p_f - p_s =$ true difference in proportions of freshmen and sophomores who purchase lunch at this school
$H_0 : p_f - p_s = 0,\ H_a : p_f - p_s > 0$

CHECK:
Random: independent random samples
10%: $60 \leq 0.10$ (all freshmen at this school) and $70 \leq 0.10$ (all sophomores at this school)
Large Counts: $60(0.731) = 43.86,\ \ 60(0.269) = 16.14,\ \ 70(0.731) = 51.17,\ \ 70(0.269) = 18.83$ are all ≥ 10.

CALCULATE:
$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_c(1-\hat{p}_c)}{n_1} + \frac{\hat{p}_c(1-\hat{p}_c)}{n_2}}} = \frac{(0.75 - 0.714) - 0}{\sqrt{\frac{(0.731)(0.269)}{60} + \frac{(0.731)(0.269)}{70}}} = 0.458$$
p -value = 0.3236

CONCLUDE:
Assuming H_0 is true $(p_f - p_s = 0)$, there is a 0.3228 probability of getting a $\hat{p}_f - \hat{p}_s$ of 0.036 or greater purely by chance. Because 0.3228 > 0.05, we fail to reject H_0 and do not have convincing statistical evidence that the proportion of all freshmen who purchase lunch at this school is greater than the proportion of all sophomores.

5. B

6. C