Lesson #31 · Topic 2F · Practice

Answer Key

1. a. Yes. Each trial has two possible outcomes, success (rolling a 5) or failure (rolling something other than a 5). The rolls are independent, the number of trials is fixed ($n = 8$) and the probability of success is constant $p = \frac{1}{6}$.

   b. Yes. Each trial has two possible outcomes, success (heart) or failure (not a heart). Although the cards are selected without replacement and thus the selections are not truly independent, the 10% condition is met because $5 \leq 0.10(52)$. The number of trials is fixed $(n = 5)$ and the probability of success is constant $(p = 0.25)$.

   c. No; the independence condition is not met since the slips are selected without replacement. The 10% condition is not met since $5 > 0.10(20)$.

2. Multiple answers possible as long as $n \leq 27$. Since Calliope is sampling students without replacement, the independence condition for a binomial variable is not met, but she can still use a binomial distribution because the sample size is less than 10% of the population $n \leq 0.10(274) = 27.4$.

3. a. X = the number of people out of a random sample of 150 who have traveled outside of the U.S. at least once. X is a binomial random variable because each trial has two possible outcomes, success (have traveled outside of the U.S. at least once) or failure (have not traveled outside of the U.S. at least once), the number of trials is fixed ($n = 150$), the probability of success is constant ($p = 0.76$), and we can treat individual observations as independent since the 10% condition is met (150 is less than 10% of all Americans).

   b. $P(X \leq 110) = \text{binomcdf}(n = 150, p = 0.76, X = 76) = 0.2488$

   c. $np = 150(0.76) = 114 \geq 10, n(1-p) = 150(0.24) = 36 \geq 10$, so the Large Counts condition for using the normal approximation is met.

   d. $\mu_X = 114, \sigma_X = 5.231$

   e. $z = \dfrac{110 - 114}{5.231} = -0.765$, Area = 0.222

4. a. The 10% condition is satisfied since the sample of 200 foods is less than 10% of all foods at Oummu's large grocery store.

   b. 54 foods

   c. 6.28 foods

   d. F is approximately normal since the Large Counts condition has been satisfied. $np = 54 \geq 10$ and $n(1-p) = 146 \geq 10$

   e. 0.0007

   f. Yes, there is convincing statistical evidence that the true proportion of foods with trans fat at her grocery store is actually higher than 0.27. If the true proportion of foods with trans fat is 0.27, the probability of getting 74 foods or greater containing trans fat in a s

5. a. For very small values of p, the distribution will be skewed right, since probabilities will be high for values of X close to 0 (a peak in the distribution), but quickly decrease for larger values of X. Probabilities for large values of X will be approximately 0.

   b. For very small values of n, the distribution may be skewed left, skewed right, or roughly symmetric but since there are relatively few possible values for X, the distribution will not have the smooth shape of a normal curve.

   c. For very large values of p, the distribution will be skewed left, since probabilities will be high for values of X close to n (a peak in the distribution) but quickly decrease for values of X further from n. Probabilities for small values of X will be approximately 0.

   d. The product np will be sufficiently large only if both n and p are sufficiently large (as shown in parts a and b). The additional condition of $n(1-p) \geq 10$ rules out very large values of p (as shown in part c) since large values of p make the quantity ($1-p$) very small.

   e. $p = 0.5$ because the expected number of successes (15) and failures (15) are both greater than or equal to 10 in this scenario, which is not the case for any of the other possibilities.

6. D

7. B