Lesson #31 · Topic 2F · Practice

Name ____________________

1. Determine if a binomial distribution can be used to model each of the following random variables. Give a reason for your answer.

   a. Rolling a fair six-sided die eight times and counting the number of rolls that land on a 5.

   b. Randomly selecting five cards from a standard 52-card deck without replacement and counting the number of cards that are hearts.

   c. Writing the numbers 1-20 on 20 separate slips of paper, putting them in a hat, and randomly selecting five of them, then counting the number of selected slips (out of 5) with an even number written on them.

2. Calliope's school recently conducted a survey among the student body and found that 39% of students at the school would prefer having year-round-school with Fridays, Saturdays, and Sundays off, rather than going to school five days a week and having a long summer vacation. There are 274 students at Calliope's school. Calliope wants to take a random sample of students from her school and count the number of students who prefer year-round school. Let Y be the number of students in a random sample of size n who prefer year-round school.

   Give three possible values of n that would allow Calliope to model Y with a binomial distribution. Explain.

3. According to a study from the Pew Research Center, 76% of Americans have traveled outside of the U.S. at least once. A random sample of 150 Americans was selected. Consider the number of Americans in the sample who have traveled outside of the U.S. at least once.

a. Define the variable of interest and describe how it is distributed.

b. Use a binomial distribution to calculate the probability that at most 110 people in the sample have traveled outside of the U.S.

c. Justify why the variable defined in part a can be approximated by a normal distribution.

d. What is the mean and standard deviation of the random variable?

e. Use a normal distribution to estimate the probability that at most 110 people in the sample have traveled outside of the U.S.

4. The Environmental Working Group (EWG) has a database with the nutritional information of more than 84,000 foods. They found that 27% of these foods contain trans fat, an industrially produced fat that is known to have particularly negative effects on cardiovascular health.

Oummu wanted to investigate how many food items contain trans fat at her large local grocery store. She selected a random sample of 200 foods from the store. Let F be the number of foods in a random sample of 200 that contain trans fat. Assume that Oummu's grocery store has the same percent of foods with trans fat as the EWG database.

a. Explain why F can be modeled with a binomial distribution, even though the foods are selected without replacement.

b. How many foods in a sample of size 200 would you expect to contain trans fat?

c. In repeated samples of size 200, by how much would you expect the number of foods containing trans fat to typically vary from the expected number calculated in part b?

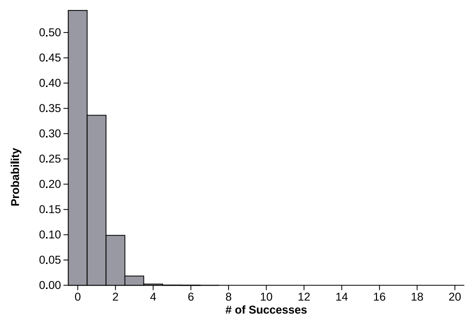
d. Describe the shape of the distribution of F . Give a reason for your answer.

e. Suppose that Oummu found 74 food items in her sample to contain trans fat. If the true proportion of foods containing trans fat is actually 0.27, what is the probability of getting Oummu's result or greater? Use a normal approximation.

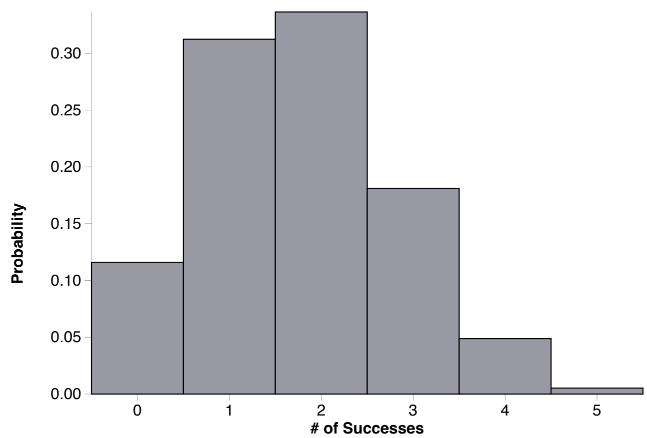
f. Does Oummu have convincing statistical evidence that the true proportion of foods with trans fat at her grocery store is actually higher than 0.27? Explain.

5. To understand why the Large Counts condition allows us to approximate a binomial distribution with a normal distribution, we will consider the distribution of some binomial random variables where the Large Counts condition is not satisfied.

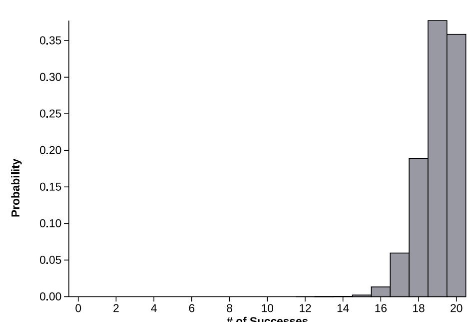
a. Consider a binomial random variable X with $n = 20$ and $p = 0.03$. The probability distribution of X is shown. Explain why a very small value of p with an n that isn't large enough gives a distribution that is not approximately normal.



b. Consider a binomial random variable Y with $n = 5$ and $p = 0.35$. The probability distribution of Y is shown. Explain why a very small value of n gives a distribution that is nonnormal.



c. Consider a binomial random variable Z with $n = 20$ and $p = 0.95$. The probability distribution of Z is shown. Explain why a very large value of p with an n that isn't large enough would give a distribution that is not approximately normal.



d. Why do you think the Large Counts condition requires that both $np \geq 10$ and $n(1-p) \geq 10$?

e. Consider a binomial random variable G with a sample size of $n = 30$. Which of the following values of p would produce a probability distribution that most resembles a normal distribution: $p = 0.01, p = 0.20, p = 0.5, p = 0.80,$ or $p = 0.99$? Explain.

6. Of the Fortune 500 companies (the top 500 US companies with the highest revenue), 10.6% are run by female CEOs . Suppose we choose a random sample of 30 companies from this list and count the number of companies in the sample that have female CEOs. Is the random variable of interest binomial with $n = 30$ and $p = 0.106$?

A) Yes, because $n(1-p) \geq 10$.

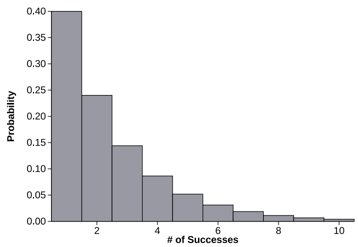
B) No, because the expected number of female CEOs in the sample is less than 10.

C) No, because the number of trials is not fixed.

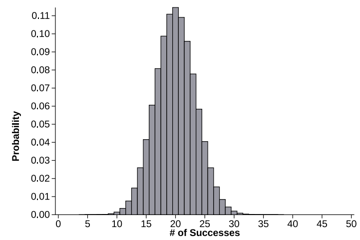
D) Yes, because the sample size is less than 10% of the population.

7. Let X be a binomial random variable with $n = 50$ and $p = 0.40$. Which of the following graphs represents the probability distribution of X ?

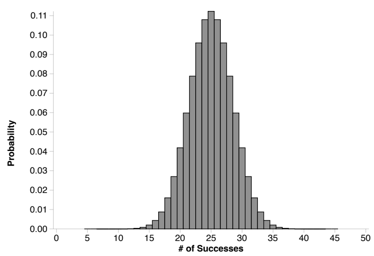
A)



B)



C)



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